

# Formally Self-Dual Codes over a Ring of Characteristic 2 and Their Binary Images

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**Abstract:** Formally self-dual codes and their construction methods have been popular among researchers in the last decade since these codes can have better parameters than self-dual codes can have. The idea of construction methods started with a few examples for binary codes found in [1] and then it is developed by numerous researchers using different rings [2], [3], [4], [5]. Some constructions for binary codes from [1] are proven to be valid for every ring of characteristic 2 in [2].

In this work, formally self-dual codes over  $\mathcal{S}_4 = \mathbb{F}_2 + u\mathbb{F}_2 + u^2\mathbb{F}_2 + u^3\mathbb{F}_2$  and some of their construction methods similar to the ones in [2] are considered. In [6] cyclic and constacyclic codes over this ring were studied based on a newly defined Gray map and Lee weight. Unlike the Gray map defined in [2] the Gray map introduced here is not duality-preserving, however the MacWilliams identities, having been proven for the Lee weight enumerators, the binary images of formally self-dual codes over  $\mathcal{S}_4$  are also formally self-dual.

Binary codes with large automorphism groups are obtained as Gray images of extremal formally self-dual codes over  $\mathcal{S}_4$  by using some computer algorithms in MAGMA computer program, which is a large, well-supported software package designed for computations in algebra, coding theory, number theory, algebraic geometry and algebraic combinatorics. We have constructed a search algorithm for large matrix groups in MAGMA that allows us to find formally self-dual codes, their Gray images and some other relevant information.

**Keywords:** Finite chain rings, Formally self-dual codes, Automorphism groups.

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